United Kingdom
Mathematics Trust

Junior Mathematical Challenge

Thursday 30 April 2020

For reasons of space, these solutions are necessarily brief.

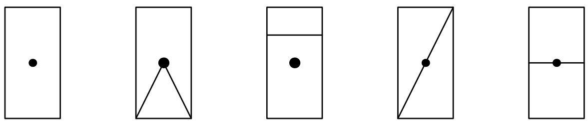
There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:

www.ukmt.org.uk

1. E It is clear that 111 is a multiple of 3 since the sum of its digits is 3. Therefore 111 is not prime.

2. D The value of $2020 \div 20$ is equal to the value of $202 \div 2 = 101$.

3. C The only rotational symmetry of the rectangle is rotation through 180° and so has order two. The diagrams below show the effect of this rotation on each of the five figures.



Only the first, fourth and fifth of these figures remain unchanged and so just three figures have rotational symmetry of order two.

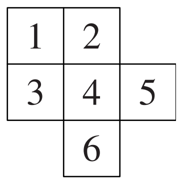
4. B There are 100 cm in 1 m. So the number of centimetres in 66.6 m is $100 \times 66.6 = 6660$.

5. E Let Amrita's number be n. The information in the question tells us that $(2n + 9) \div 3 - 1 = n$. Therefore $2n + 9 = 3(n + 1) = 3n + 3$. Hence $n = 9 - 3 = 6$.

6. C The value of $\frac{6}{12} - \frac{5}{12} + \frac{4}{12} - \frac{3}{12} + \frac{2}{12} - \frac{1}{12} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$.

7. A Note that $110 = 10 \times 11 = 2 \times 5 \times 11$, which is the prime factorisation of 110. Therefore the four different positive integers whose product is 110 are 1, 2, 5 and 11. Their sum is 19.

8. D The diagram shows the original diagram with the cells labelled 1, 2, 3, 4, 5, 6. The pairs of cells which Wesley can colour are 1 and 4; 2 and 3; 2 and 5; 3 and 6; 5 and 6. Therefore the diagram can be coloured in five ways.



9. A Let the required number be n. Then, $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times n = 2$. Therefore $\frac{1}{120} \times n = 2$. Hence $n = 120 \times 2 = 240$.

10. D The solutions of the four equations are, from left to right, $x = 12$; $x = 48$; $x = 8$; $x = 13$. Therefore exactly one of the equations has solution $x = 12$.

11. **E** In a 20 by 20 grid there are 21 horizontal wires each of length 20 cm and 21 vertical wires each of length 20 cm.
Therefore the total length of wire required for a 20 by 20 grid is $42 \times 20\,\text{cm} = 840\,\text{cm}$.

12. **B** The large equilateral triangle has been divided into four congruent equilateral triangles.
Therefore each of these triangles has an area equal to one quarter of the area of the large triangle.
Similarly, the equilateral triangle which is partly shaded has been divided into four congruent equilateral triangles, three of which are shaded.
Therefore the fraction of the area of the large triangle which has been shaded is $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$.

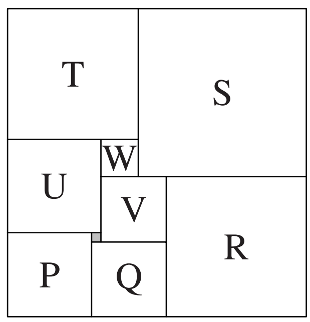
13. **B** The mean of four positive integers is 5. Therefore the sum of the four integers is $4 \times 5 = 20$.
The median of the integers is the mean of the two middle integers.
Since this median is 6, the sum of the two middle integers is $2 \times 6 = 12$.
Hence the sum of the smallest and largest of the four integers is $20 - 12 = 8$.
Therefore the mean of the largest and smallest of the integers is $8 \div 2 = 4$.

14. **D** Let the size, in degrees, of $\angle OLM$ be $2x$. Then, in degrees, the size of $\angle PON$ is x.
Therefore, since vertically opposite angles are equal, $\angle KOL = x°$.
The sum of the angles on a straight line is 180°, so $\angle LKO = 180° - 124° = 56°$.
An exterior angle of a triangle is equal to the sum of the two opposite, interior angles.
Therefore, in triangle KOL, $2x = x + 56$. So $x = 56$.
Hence $\angle OLM$ is $2 \times 56° = 112°$.

15. **E** Let the number of children who play only football be f, the number of children who play only tennis be t and the number of children who play both sports be b.
Since there are 42 children, $f + t + b = 42$.
Also, since the number of children who play tennis is equal to the number of children who play only football, $t + b = f$. Therefore $f + f = 42$. So $f = 21$ and $t + b = 21$.
Finally, twice as many play both tennis and football as play just tennis. Therefore $b = 2t$.
Substituting for b, gives $t + 2t = 21$. Hence $t = 7$.
Therefore the number of children who play football is $42 - t = 42 - 7 = 35$.

16. **A** Note that $\frac{6}{25} = \frac{24}{100} = 0.24$. Therefore $\frac{6}{25}$ cannot be made from the sequence "0625".
Of the other four options, $\frac{5}{8} = 0.625$; $\frac{1}{16} = .0625$; $\frac{25}{4} = 06.25$; $25^2 = 0625$.
Hence $\frac{6}{25}$ is the only option which cannot be made from the given sequence.

17. **E** Label the individual squares, except for the smallest, P, Q, R, S, T, U, V and W as shown. The side-length of the smallest square is 1. Since the next smallest squares are W and V, they have side-lengths 4 and 7 respectively. Therefore Q has side-length $7 + 1 = 8$, P has side-length $8 + 1 = 9$ and U has side-length $9 + 1 = 10$. Hence T has side-length $10 + 4 = 14$ and R has side-length $7 + 8 = 15$. Therefore the rectangle has side-lengths $9 + 8 + 15 = 32$ and $9 + 10 + 14 = 33$. Therefore the area of the rectangle is $32 \times 33 = 1056$.



(*Alternatively: the three largest squares R, S and T have side-length 15, 18 and 14 respectively. So the rectangle has sides of length 18 + 15 and 18 + 14 and so measures 33 by 32.*)

18. **B** The digits which are non-prime are 0, 1, 4, 6, 8, 9. However, the units digit of a prime cannot be 0, 4, 6 or 8.
Therefore any two-digit primes which have both their digits non-prime have a units digit of 1 or 9.
The only such primes are 11, 19, 41, 61 and 89. Hence there five such primes.

19. **E** Adding the top row and the middle column gives,
$2J + K + 2K + J = 5 + 7 = 12$. Hence $3J + 3K = 12$. So $J + K = 4$.
The first column shows that $J + K + L = 11$.
Hence, $J + K + L - (J + K) = 11 - 4 = 7$. Therefore $L = 7$.
(*It is then possible to deduce that $J = 1$ and $K = 3$ and check that each total is correct.*)

J	K	J	5
K	K	L	13
L	J	L	15
11	7	15	

20. **D** Edmund makes a cube using eight small cubes. Therefore his cube measures $2 \times 2 \times 2$.
Since Samuel makes a cuboid twice as long, three times as wide and four times as high as Edmund's cube, Samuel's cube measures $4 \times 6 \times 8$. Therefore Samuel uses 192 small cubes in making his cube.
Hence Samuel uses $192 - 8 = 184$ more small cubes than Edmund.

21. **D** It is possible to eliminate certain options by comparing the units digit of the original product with the units digit of the new product.
The units digit of 25×36 is 0, but the units digit of 52×63 is 6. So A is not the correct option.
The corresponding units digits for the other options are B: 8 and 2; C: 4 and 5; D: 6 and 6; E: 6 and 2. These calculations suggest that the correct option is D.
In confirmation, note that $42 \times 48 = 2 \times 21 \times 2 \times 24 = 24 \times 2 \times 2 \times 21 = 24 \times 84$.

22. **E** Let the side-length of Harriet's square be $4x$ cm. Then, the rectangle obtained when the square is folded in half measures $4x$ cm by $2x$ cm.
When that rectangle is folded in half, it gives a rectangle which is not a square so this rectangle measures $4x$ cm by x cm. The perimeter of the smaller rectangle is $2(4x + x)$ cm $= 10x$ cm.
Therefore, since the perimeter of the smaller rectangle is given as 30 cm, we have $x = 3$.
Hence the side-length of Harriet's square is 12 cm and its area is 144 cm^2.

23. **B** The four smallest primes are 2, 3, 5, 7. Since these are all prime, their lowest common multiple is $2 \times 3 \times 5 \times 7 = 210$.
Therefore the smallest positive integer which leaves remainder 1 when divided by each of the four smallest primes is $210 + 1 = 211$. The next positive such integer is $2 \times 210 + 1 = 421$.
Therefore the required difference is $421 - 211 = 210$.

24. **B** For the number of adjacent empty chairs in a single row to be a maximum, all rows apart from that one must hold as many parents as possible.

P	P	*	P	P	*	P	P	*	P
P	P	*	P	P	*	*	*	*	P

In view of Susan's observation, the maximum number of parents sitting in any row is seven, as the upper diagram shows. Therefore, the maximum number of parents in seven of the eight rows is $7 \times 7 = 49$. The number in the remaining row is then $54 - 49 = 5$. In that case, and again in view of Susan's observation, the maximum number of adjacent empty chairs would be four as shown in the lower diagram.

25. **E** Let $a°$, $b°$, $c°$, $d°$ and $e°$ be the sizes of the angles shown in the diagram. Since angles on a straight line add to 180°, the points Q and R on line PS give the equations

$x + 2y + a = 180$ and $y + 2x + b = 180$.

Adding these two equations gives $3x + 3y + a + b = 360$.

The sum of the interior angles of a triangle is 180°.

Hence, in triangle QMR, $a + b + 33 = 180$.

Subtracting this equation from the previous one gives

$3x + 3y - 33 = 180$ and hence $x + y = 71$.

Since vertically opposite angles are equal, we have $d = 2y$ and $e = 2x$.

Also, from triangle QRK we see that $d + e + c = 180$.

Therefore $2x + 2y + c = 180$ and hence $142 + c = 180$.

So the size of angle JKL is 38°.